An Aerodynamic Method for Sizing Sands and Other Granular Materials[1]

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Abstract

A method of separating granular materials according to size by virtue of their relative displacements when dropped from rest into a moving air stream is presented. The equations of motion for a spherical particle in a moving air stream were solved using an analog computer. This method of solution allowed the drag forces in both the horizontal and vertical directions to be evaluated in a relatively easy manner. The solution was used to design an apparatus to separate granular materials in the sand size range. The apparatus was constructed and tests were conducted to determine the trajectories of various particle sizes for a range of air velocities. The results were compared with those predicted by the analog solution. The agreement of the predicted and observed trajectories and the reliability with which the apparatus will size samples of glass beads and sands are discussed. The apparatus will be useful for sizing large quantities of granular materials for use in mechanics of erosion, infiltration and overland flow studies.

Introduction

Sands, glass beads, or other small granular materials satisfying narrow size range requirements are often needed by the researcher conducting experimental work in soil science, geology, engineering or related areas. These materials are usually difficult to obtain, especially when large quantities are needed for studies such as the mechanics of soil erosion and flow of fluids through porous media. The conventionally used method of sizing the materials by sieving techniques is a slow process requiring that the material be passed through a number of sieves of decreasing opening sizes. Often the desired size range cannot be obtained by available standard sieves.

Aerodyamic methods have been used commercially for several years in the separation of chaff, weed seeds and other undesirable materials from grain and seeds. Muller, et al. (3) designed an apparatus to sort walnuts on the basis of their different aerodynamic properties. Meyer (2) used an aerodynamic method to size glass beads, sands, and carborundum.

This paper describes an apparatus which separates, according to size, small granular materials in the sand size range by virtue of their horizontal displacement when dropped from rest into a horizontally moving air stream. Analytical solutions of the equations of motion for a particle in a moving air stream are presented. Application of these solutions to the design of the separator and results of experiments for determining the reliability with which the separator can be used to size glass beads are discussed.

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Theory

The first step in the design of the separator was the derivation of the equations of motion for a particle in a uniformly moving air stream. Consider a particle released from rest into a horizontal air stream

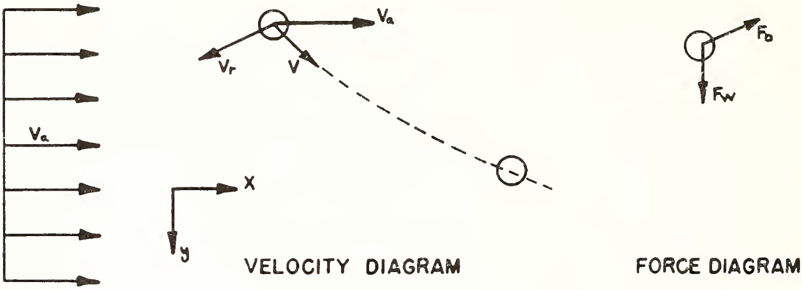

Figure 1. Velocity and Force Diagrams for a Particle in a Horizontal Air Stream.

(Figure 1). At any point along its trajectory the particle has a tangential velocity (V). The relative velocity (V_r) between the particle and the air is the vector difference between these two velocities. The x and y components of the relative velocity may be written as follows:

$$V_{rx} = V_x - V_a \tag{1}$$

$$V_{ry} = V_y \tag{2}$$

Then V_r may be evaluated

$$V_r = ((V_x - V_a)^2 + V_y^2)^{1/2} \tag{3}$$

There are two forces acting on the particle, the weight (F_w) acting in the downward (+y) direction and the drag force (F_d) acting in a direction opposite to V_r. The equations of motion for the particle are obtained in terms of these forces by applying the second law of motion:

$$\frac{d^2x}{dt^2} = -\frac{V_{rx}}{MV_r} F_d \tag{4}$$

$$\frac{d^2y}{dt^2} = \frac{1}{M} \left(F_w - \frac{V_{ry}}{V_r} F_d\right) \tag{5}$$

in which F_w and F_d are

$$F_w = Mg \tag{6}$$

$$F_d = DA\rho \frac{V_r^2}{2} \tag{7}$$

and where x, y = coordinates of the particles position
M = mass of the particle

A = projected area of the particle on a plane perpendicular to the direction of motion
D = the drag coefficient as defined by Vennard (4)
ρ = density of the fluid
t = time
μ = viscosity of the fluid
g = acceleration due to gravity

For spherical particles the relationship between the drag coefficient and the Reynolds number (R) is well known, e.g. Bird, et al. (1). Preliminary calculations showed that the Reynolds number range for the intended application was $1 \leqslant R \leqslant 100$ with the particle diameter as the characteristic length. It was also determined that this range can be approximated within 2% by the equation

$$D = \frac{K}{R} \qquad (8)$$

where $K = aR + b$ and the parameters a and b are step functions of R having the following values:

	a	b
$1 \leqslant R \leqslant 10$	1.889	22.01
$10 \leqslant R \leqslant 30$	0.95	31.5
$30 \leqslant R \leqslant 60$	0.80	36.0
$60 \leqslant R \leqslant 100$	0.65	45.0

When $R \leqslant 1$, $K = 24$; hence for all values of $R < 1$, Equation *7* reduces to the familiar Stoke's law for creeping flow around a sphere.

By using the defining equation for Reynolds number and evaluating A and M in equations *6* and *7* in terms of the radius of the sphere (r) and its density (P), and by substituting the resulting relationships along with Equation *8* into Equations *4* and *5*, the equations of motion for a spherical particle in a moving air stream may be written as follows:

$$\frac{d^2x}{dt^2} = \frac{3K\mu}{16r^2P\,[V_a - dx/dt]} \qquad (9)$$

$$\frac{d^2y}{dt^2} = g - \frac{3K\mu}{16r^2P}\frac{dy}{dt} \qquad (10)$$

Solution of the Equations of Motion

The next step in the design of the separator was the solution of Equations *9* and *10* subject to initial conditions of $x = y = 0$ and $\frac{dx}{dt} = \frac{dy}{dt} = 0$. Equations *9* and *10* were programmed on an EAI TR-48 analog computer as shown in Table 1 and Figure 2. The constant input data were the values of the particle density (P) and the viscosity of the air (μ). Solutions in the form of plots of the trajectories of

TABLE 1

Amplifier Outputs and Potentiometer Settings for the Analog Computer Solution

Amplifier No.	Output	Pot. No.	Setting
02	$-\dot{x}$	00	SF (Scale Factor)
03	x	01	SF
		02	V_a
04, 05	$V_a - \dot{x}$		
06	$-\frac{1}{r^2}(V_a - \dot{x})$	03	$\frac{1}{r^2}$
07	$\frac{3\mu 1}{16\,P\,r^2}(V_a - \dot{x})$	04	$\frac{3\ \mu}{16\ P}$
08	$\frac{3\mu K}{16\,P\,r^2}(V_a - \dot{x})$	08	SF
		15	SF
15	$(V_a - \dot{x})^2 + \dot{y}^2 = V_r^2$	17	r
16	$-\frac{\rho}{\mu} V_r$	18	a_1 Constants for
17	$\frac{2r\rho}{\mu} V_r$ = Reynold's number	20	b_1 evaluation of K
		21	a_2 for different
		22	b_2 Reynold's number

18	$\dot{y}$	23	a_3 ranges
19	$\dfrac{-\dot{y}}{y}$	25	b_3
20	$\dot{y}$	26	Comparator references for the Reynold's number ranges
21	y^2	27	
22	$\dfrac{-y}{r^2}$	30	g, acceleration due to gravity
23	$\dfrac{3\mu 1}{16\,P\,r^2}\dot{y}$	31	SF
29	K for Different	32	$\dfrac{1}{r^2}$
30	Ranges of Reynold's		
31	number	33	$\dfrac{3\ \mu}{16\ P}$
A	$-(V_a - \mathring{x})^2$		
B	$+V_r$	35	SF
C	$-\dfrac{3\ \mu K}{16\,P\,r^2}\dot{y}$	36	SF

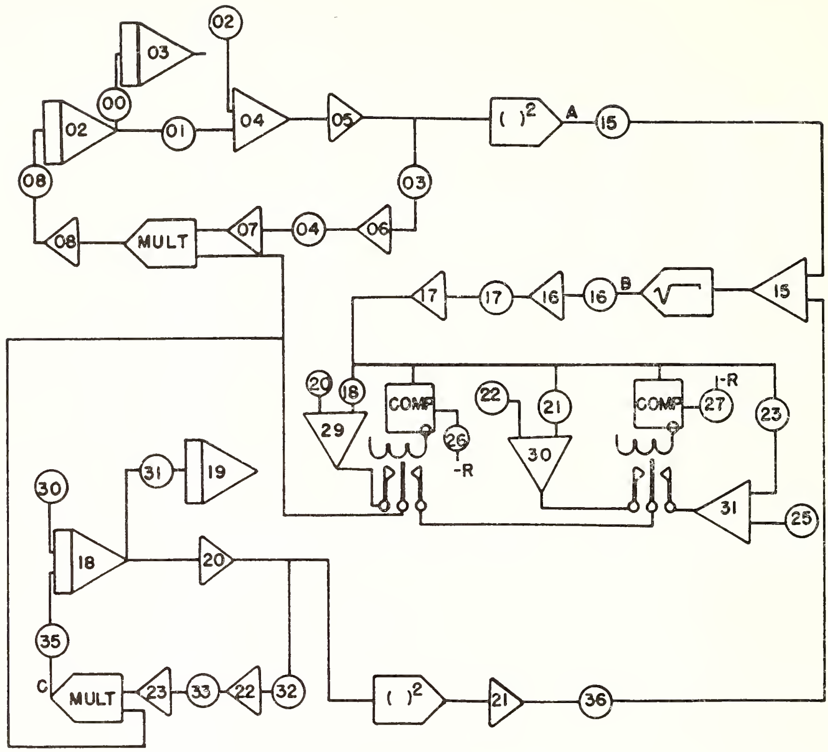

FIGURE 2. A SCHEMATIC DIAGRAM OF THE ANALOG COMPUTER SOLUTION OF THE EQUATIONS OF MOTION FOR A SPHERICAL PARTICLE IN AN AIR STREAM

the particles for a range of air velocities and particle radii were obtained by simply changing the potentiometer setting corresponding to these values in the analog program. Solutions for six glass bead sizes at two different air velocities are shown in Figure 3.

The Separator

Solutions were obtained for a wide range of air velocities to determine the dimensions of a separator to size material with diameters less than 500 microns. A separator was designed and constructed on the basis of these solutions. A schematic diagram of the separator is shown in Figure 4 and an overall pictorial view in Figure 5.

To meet the conditions under which the analytical solutions were obtained, the separator was designed such that the velocity profile in the separator duct was uniform and non-turbulent. This condition was best obtained with a suction system. A centrifugal blower was used to pull air through the separator duct into a plenum where a constant negative pressure was maintained. The blower had an inlet diameter of 10.75 inches and was powered by a ½ HP, 2 speed electric motor. A variable speed pulley arrangement allowed a velocity range in the

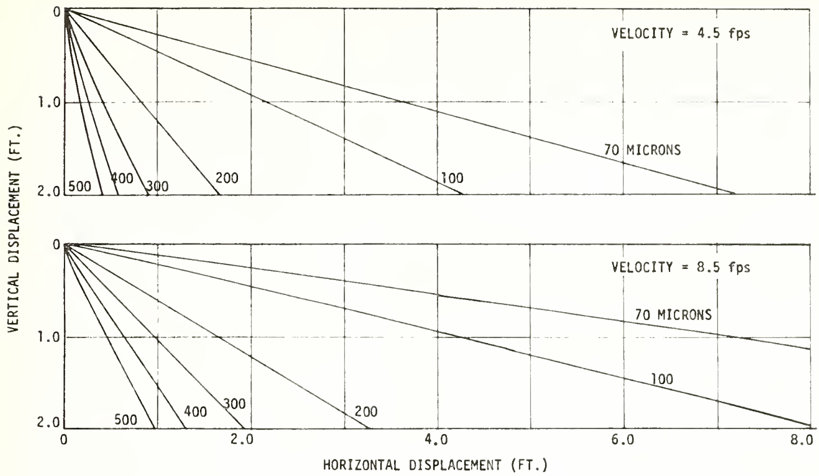

FIGURE 3. THEORETICAL TRAJECTORIES OF SPHERICAL PARTICLES IN A HORIZONTAL AIR STREAM

separator duct of 2.5-20.0 fps. The blower RPM was calibrated to the air velocity in the duct. The velocity range could be extended downward to zero fps by partially closing the outlet of the exhaust blower.

To retard boundary layer buildup at the entrance, an entrance section consisting of 4 cylindrical sections with radii of 6 inches was used. Partitions were placed at 3 inch intervals in the bottom of the separator duct forming 30 collection hoppers. A sliding gate at the front of each hopper allowed its contents to be easily emptied. One side of the separator duct was constructed of a transparent plastic sheet to enable the operator to view the trajectories of the particles

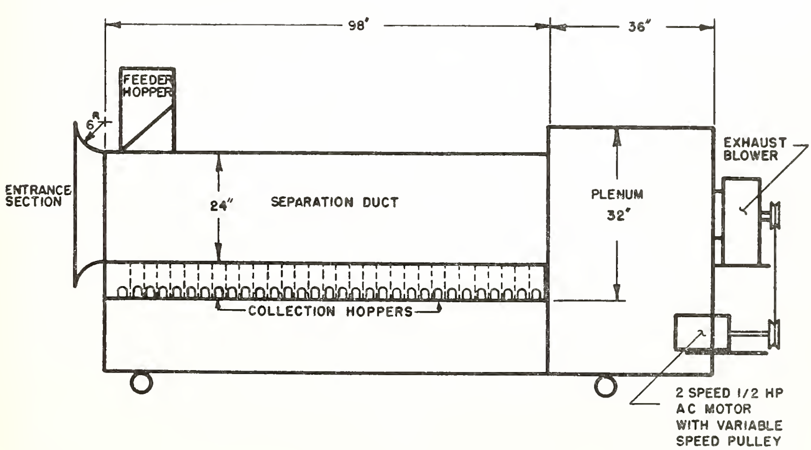

FIGURE 4. A SCHEMATIC DRAWING OF THE SEPARATOR

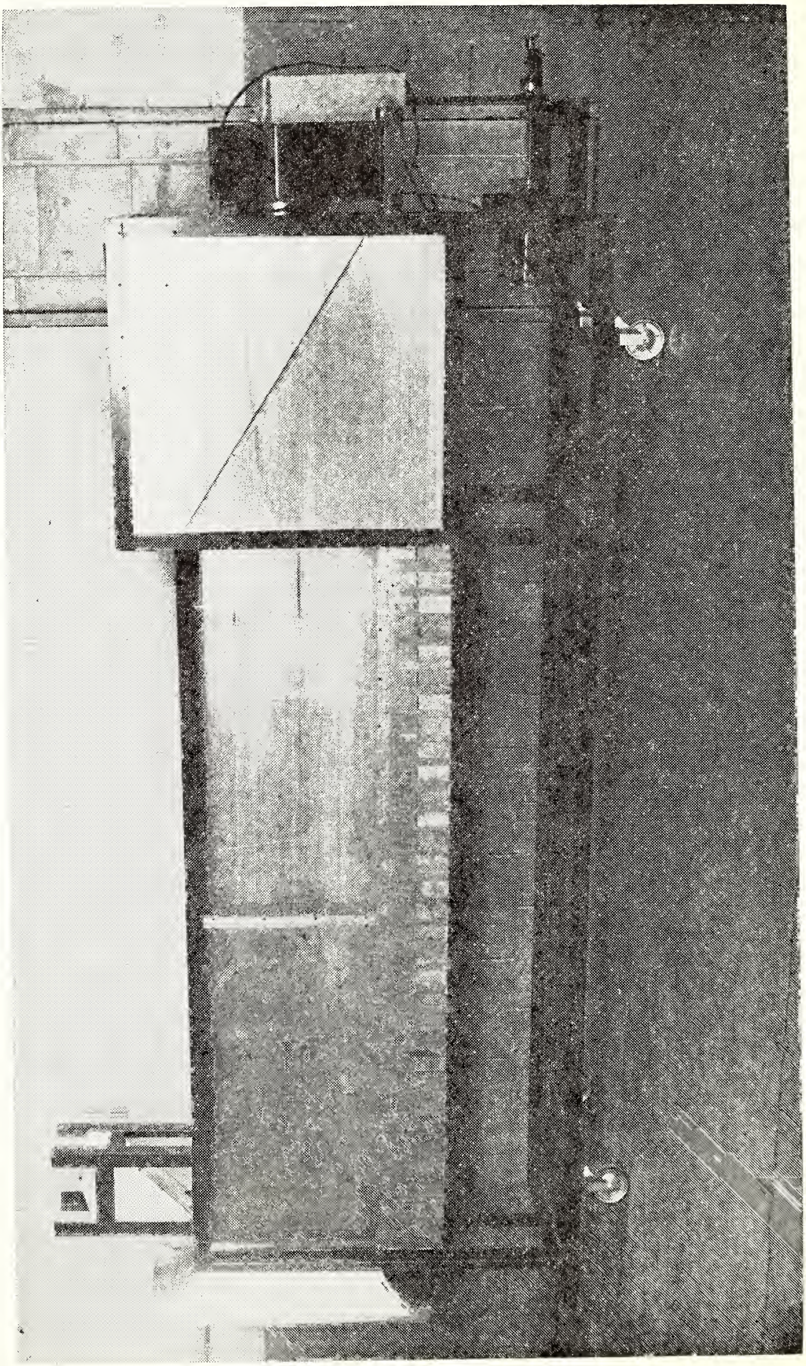

Figure 5. Front view of the separator.

and the filling of the hoppers. The particles were introduced into the separator by a feeder hopper which allowed a thin stream of particles to fall through a slit in the top of the duct. An adjustable opening controlled the rate of inflow. Since the separator was designed on the basis of solutions for a single particle, inflow rates large enough to cause interference between particles would obviously limit the efficiency of separation obtained. Satisfactory separation was obtained, however, with a slit width of approximately three times the diameter of the largest particle.

Evaluation of Performance

Velocity profiles measured for centerline velocities of 5 and 10 fps showed that, at the point of particle entry, a uniform velocity was obtained 0.4 and 0.3 inches, respectively, below the top of the duct. Substituting the measured velocity profiles into the analog computer program proved that boundary layers of this size had a negligible effect on particle trajectories. Although quantative measurements of turbulance in the duct were not made, observations using smoke and dust indicated turbulence along the bottom of the duct. This turbulance was primarily due to the hopper partitions. In preliminary tests glass beads that were displaced more than 5 feet from the point of entry tended to "float" due to the turbulence and to the flatness of their trajectories. Separation occurring beyond this point was not considered reliable.

The degree of separation expected from a single pass through the separator is shown in Figure 3. Good separation can be obtained for particles ranging in size from 100 to 200 microns using a velocity of 4.5 fps. For particles in the 200 to 400 micron range, much better separation can be obtained using a velocity of 8.5 fps. However, if this velocity was used for the smaller particles they would be carried into the region where turbulence becomes important resulting in inadequate sizing. Therefore, if the sample to be sized has a wide range of particle diameters, a multipass procedure is desirable.

Reliability of Separation

The limits of accuracy with which glass beads can be sized at a given velocity were determined theoretical and compared to observed values. Samples of three different bead sizes were passed through the separator with an air velocity of 10 fps. The beads which were collected in hoppers at 15 inches, 22 inches, and 28 inches from the point of entry were passed through the separator a second time at the same velocity. Then samples were taken from each of the three hoppers. Microscopic determinations of the diameters of 50 beads selected at random were made, and their means and standard deviations computed. Solutions for the mean diameter and for the range of diameters which would theoretically fall in each of the three hoppers were also determined. Assuming that the samples had uniform distribution of particle sizes, the theoretical standard deviation for each hopper was computed and compared to the value for the observed data. A summary of the results is presented in Table 2.

TABLE 2

Comparison of Predicted and Actual Displacements
For Three Sizes of Glass Beads

Mean Diameter (microns)	Standard Deviation (microns)		Displacement (inches)	
	Observed	Theoretical	Actual	Theoretical
472	12.4	12.2	14-17	14.6-17
245	6.5	5.8	32-35	34 -38
225	5.8	5.8	38-41	38 -42

An envelope of the trajectories of beads with a mean diameter of 472 microns was drawn on the transparent front of the separator during a run. The theoretical trajectories for the maximum and minimum sizes collected and the envelope of the observed trajectories are compared graphically in Figure 6.

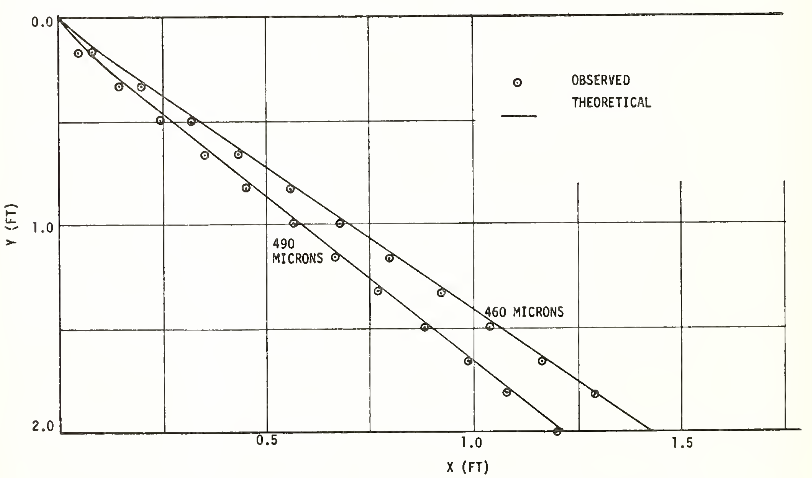

FIGURE 6. THEORETICAL AND OBSERVED TRAJECTORIES FOR A VELOCITY OF 10 FT./SEC.

The reliability with which the separator could size samples of glass beads with a large range of diameters was determined. Approximately 4 gallons of glass beads ranging in diameter from 50 to 525 microns were sized with the separator. Two passes through the separator were required for this size range. The first pass was made at an air velocity of 4.5 fps. Samples of beads were taken from each of the hoppers and the diameters of 25 randomly selected beads were determined microscopically. The beads which were displaced less than 2 feet were mixed and rerun at a velocity of 8.5 fps. Samples were again taken and microscopic measurements made as before. The mean diameter and standard deviation were determined for the beads in each of the hoppers. Theoretical values for the displacements were obtained from the analog solutions and, assuming a uniform distribution of particle sizes, theoretical standard deviations were calculated. Results shown in Table 3 indicate beads with diameters less than 200 microns were adequately sized with an air velocity of 4.5 fps. A higher air velocity (in this case, 8.5 fps) was needed with larger diameter beads.

The summary in Table 3 shows that the coefficients of variation were on the order of 6 to 11 percent for bead sizes less than 200 microns and 4 percent for bead sizes greater than 200 microns. A notable exception was the 275 micron bead size where the coefficient of variation was 11 percent. This relatively high coefficient of variation was caused by a nonuniform distribution of particle sizes in the original sample. Beads in the two adjacent hoppers were of the same mean diameter indicating that the sample contained a large number of beads with diameters around 240 microns and relatively few beads between 250-360 microns. A similar circumstance occurred with beads having a mean diameter of 128 microns. A discontinuity between bead sizes of 130 and 100 microns existed giving a small fraction with a mean diameter of 128 microns and, hence, a relatively large coefficient of variation.

It was concluded that in general the degree of separation and the coefficients of variation obtained were acceptable for most purposes.

TABLE 3

Summary of Data for Sizing Glass Beads

Mean Diameter (microns)	Standard Deviation Observed (microns)	Standard Deviation Theoretical (microns)	Coefficient of Variation Observed (%)	Coefficient of Variation Theoretical (%)	Horizontal Displacement Observed (inches)	Velocity (fps)
500	19	22	3.8	4.4	8-11	8.5
455	20	18.5	4.4	4.0	11-14	8.5
410	15	14.4	3.7	3.5	14-17	8.5
360	15	11.5	4.2	3.2	17-20	8.5
275	30	10.0	11.0	3.6	20-23	8.5
244	10	14.4	4.1	5.7	23-29	8.5
225	5	5.8	2.2	2.6	29-32	8.5
217	10	4.9	4.6	2.3	32-35	8.5
207	9	3.8	4.4	1.8	35-38	8.5
200	8	3.5	4.0	1.7	38-44	8.5
176	11	4.9	6.3	2.9	38-44	4.5
144	10	4.6	7.0	3.2	20-23	4.5
134	5	3.8	3.7	2.8	23-26	4.5
128	14	7.8	11	6.1	29-38	4.5
98	11	2.0	11	2.0	38-41	4.5
86	10	1.7	11.6	2.0	41-44	4.5
75	7	1.7	10	2.3	44-47	4.5

They would compare favorably to those which could be obtained with standard sieving techniques.

Summary

An apparatus was designed and constructed to use an aerodynamic method to size granular materials in the sand size range. The material was sized by virtue of the relative displacements of particles of different diameters when dropped from rest into a horizontal air stream.

The equations of motion for a spherical particle in an air stream were derived and an analog computer was programmed to solve these equations for wide ranges of air velocities and particle diameters. The design of the separator was based on these solutions. Glass beads were used in experiments to determine the reliability with which the separator could size materials. The results of the experiments showed a close correspondence between observed trajectories and displacements of the particles and those values predicted by the analog computer solutions. A sample of beads ranging in diameter from 50 to 525 microns was sized using the separator. The results favorably compared to those that could be obtained using the slower, conventional sieving techniques.

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