MATHEMATICS

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ABSTRACTS

Symposium: Computers and the Collegiate Mathematics Program. Paul T. Mielke, Wabash College.—Financial problems of the small (600-1200) privately endowed liberal arts college make it unlikely that many such institutions will engage actively in the use of computers. The purchase or rental of a computer cannot be justified by its use as a teaching tool. Therefore schools engaging computers must finance them through research or special gifts.

The basic curriculum of the small college (usually consisting of about 18 hours of analysis including differential equations, 6 hours of algebra including matrix algebra, 6 hours of statistics and 12 hours of additional material such as geometry, topology and function theory) offers little possibility for addition of special courses such as numerical analysis or computer programming. However, the opportunities of orienting such courses as statistics, matrix algebra and differential equations toward computer techniques must not be overlooked.

Teachers should seek help actively from the computer manufacturers and machine users in developing the interests of their students through computer-oriented courses and extracurricular activities. Over the last two years at Wabash College an experiment involving study of the IBM 704 has met with some success.

Finally the teacher should be alert to the many opportunities for summer employment which will develop his computer interests and those of his students.

Seymour Parter, Indiana University.—Most discussions of the computers and the collegiate mathematics program seem concerned with the expansion, or inclusion, of traditional courses in numerical analysis and/or the significance of "programming" and "computer science." Little, if any, attention has been focused on the possible repercussions of the need for more sophisticated mathematics by research workers using computers. As the abilities of the machines become greater the usefulness of many mathematical subjects which are usually considered "high-brow" will become more and more apparent. Consequently, it is reasonable to expect that we will soon see the time when courses in Real Variables, Modern Algebra, Logic, Probability, Orthogonal Polynomials and Functional Analysis are regularly taught in the graduate school to large groups of non-mathematics majors. When such courses are part of the "service" obligations of the graduate mathematics curriculum, we may expect some very interesting changes in the Collegiate Mathematic program.

Dr. Kaj L. Nielsen, Allison Division, GMC.—The influence of computers upon the collegiate mathematics program should be divided into four categories:

I. The effect on the engineering mathematics program
II. The effect on the program for the industrial mathematician
III. The effect on the program for the mathematics teacher
IV. The effect on the program for the statistician.

Let us consider each in turn.

I. In addition to the standard engineering mathematics curriculum the extensive use of the computers in the solution of engineering problems now demands:

a) A course in numerical analysis
b) A course in mathematical modeling of engineering problems
c) A basic course in computer programming with laboratory work.

II. For the industrial mathematician the demands are the same as those for the engineering program plus the addition of

a) An advanced course in numerical analysis
b) A detailed course on a specific computer.

III. For the mathematics teacher the demand is for

a) An elementary course in numerical analysis
b) Enrichment of the present courses by greater emphasis on emperical equations and more detailed study of special functions.

IV. For the statistician the demand is for

a) A course in numerical analysis
b) A course in linear and dynamic programming
c) A course in computer programming with laboratory work.

It is essential that the enlarged program be kept within the mathematics department.

Some Advantages of Riemann-Stieltjes Integration. GREGERS L. KRABBE, Purdue University.—In the course of the last three decades, Lebesgue integration (i.e., integration with respect to countably-additive measure) has assumed a predominant role. However justified this may be, Lebesgue integration is inadequate in some important cases that require Riemann-Stieltjes integration.

Uncertainty and Entropy. J. H. ABBOTT, Purdue University.—The usual information theory concepts of entropy and uncertainty are introduced. Modern conditional expectation is then discussed and applied to conditional entropy and uncertainty. For example, if π is a countable and measurable partition of the space Ω in the probability space (Ω, F, P), and α is a σ-subalgebra of the σ-algebra F, then the conditional uncertainty $U(\pi|\alpha)$ of π given α is defined at $\omega \epsilon \Omega$ by

$$U(\pi|\alpha)(\omega) = -\log_2 P(A|\alpha)(\omega)$$

where A is the unique set in π for which $\omega \epsilon A$, and $P(A|\alpha)$ is some fixed version of the conditional probability of A given α. Some elementary results in information theory are then shown to follow easily using the techniques of conditional expectation.

On Generalized Functions—A Survey. MICHAEL GOLOMB, Purdue University.—The role of improper or singular functions in mathematical physics and engineering is pointed out. In particular, Green's functions, impulse functions, weak derivatives, weak solutions of partial differential equations, generalized Fourier integrals etc. are discussed. The theories of "generalized functions" or "distributions" of Soboleff, L. Schwartz, Mikusinski, and simplifications due to Temple, Lighthill, Korevaar and others are analyzed and compared. There are four main procedures in the construction of generalized functions:

A. Weak (sequential) completion of the class of continuous functions, C, weak convergence being defined by the use of integrals of the products of these functions by indefinitely differentiable functions with compact supports.

B. Strong (sequential) completion on each compact subset K of the k-th (k depending on K) primitives of the functions in C.

C. Generalized functions ("distributions") as linear functionals continuous on the space of indefinitely differentiable functions with compact supports, with a certain convergence concept (for sequences).

D. Generalized functions as quotients in the field constructed from the linear algebra with no zero divisors of the continuous functions of one variable t vanishing for $t < o$, the product being the convolution product.

A., B., C., lead to equivalent concepts. D. is especially well suited for an operational calculus a la Heaviside.

A New Introduction to the Ideas and Methods of Trigonometry. ROBERT J. THOMAS, DePauw University.—This new mathematical system was developed to illustrate the ideas and methods of trigonometry to college students without using sine's or α's which might remind them of triangles and angles (from hearsay or partial courses in high school) and without any possible distracting practical applications. The actual development of this system could also be used on the one hand to illustrate to the teacher one good method of organizing the presentation of trigonometry; or, on the other hand, to illustrate to the student the logical development of a system per se, irrespective of any analogies to trigonometry.

A New Look at Least Squares. RICHARD DOWDS, Butler University.—The modern theory of approximation is having a profound influence on numerical analysis. Not only does approximation theory give new results and methods in numerical analysis, but it also tends to group under one heading a large number of problems that were heretofore disconnected. It is the purpose of this paper to show how approximation theory can be used to bear on the ancient technique of least squares. It is the author's opinion that this new approach is more elementary than the traditional development of least squares.

A Definition for Polynomials on Topological Groups, and Some Examples. F. W. CARROLL, Purdue University.—The following definition is given by Domar: Let f be a real-valued function on a topological group G. Then f will be called a polynomial on G if (P1), (P2), and (P3) are satisfied.

(P1) For each pair x_1, x_2 of elements in G, the set of values $f(x_1 + nx_2)$ $(n = 0, \pm 1, \ldots)$ coincide with the values assumed at the integers by some polynomial on the real line.

(P2) If $d(x_1, x_2)$ denotes the degree of $f(x_1 + n x_2)$ (as a polynomial in n), then $\sup_{x_1, x_2 \epsilon G} d(x_1, x_2)$ is $< \infty$.

(P3) f is continuous.

Proposition 1: Let f be a real-valued function on the m-dimensional Euclidean space R^m, such that f satisfies (P1) and (P3). Then f is an ordinary polynomial on R^m.

Proposition 2: There are real-valued functions on R^1 which satisfy (P1) and (P2) but not (P3).

Proof: Any non-measurable additive function satisfies (P1) and (P2).

Proposition 3: Let C denote the additive group of integers. There exists a real-valued function f on the discrete group C + C such that f satisfies (P1) but not (P2).