PHYSICS

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Fourier Synthesizer

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Fourier series, from Fourier's theorem that any periodic function may be represented as an infinite sum of sinusoidal terms, are useful in many fields of science. It is desirable, therefore, to have a practical teaching aid and an industrial research tool in the form of a Fourier series synthesizer.

Industrial research can utilize a synthesizer in such fields as electronic circuit design, medical research, radar design, and vibration analysis and correction. A leading tire manufacturer, for instance, eliminated the harsh noise from one of its models by designing it so that the audio frequency noise from the tire hitting the pavement blended in with the background noise.

For classroom use and in many industrial applications, a synthesizer should be simple to operate, accurate, inexpensive, and its output should be easy to display.

Heretofore, only specialized synthesizers have been built. Lord Kelvin in 1872 built a successful tide predictor based on harmonics whose periods and phases were deduced from past records, For each harmonic he used a Scotch yoke, shown in Fig. 1. This is a device for changing rotary into linear motion.

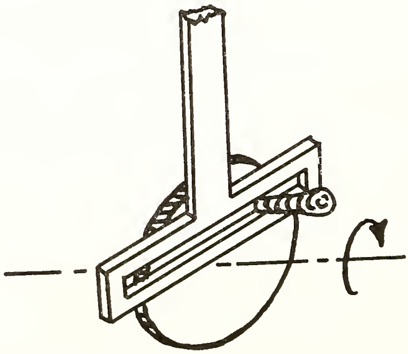

Figure 1. Scotch yoke.

One of these Scotch yokes was used for each component and the various tidal constituents (due to the moon, sun, etc.) were summed by a rope passing over pulleys connected to the ends of the Scotch yoke bars. The resultant curve was drawn on a revolving drum.

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Electrical Methods of Synthesis

Kelvin's synthesizer and several like it are for specific purposes only and are too bulky for classroom use. It is desirable, therefore, to have a smaller synthesizer, yet one which gives an electrical output with at least the first ten harmonics. An electrical output makes it convenient to display the resultant waveform on an oscilloscope.

Two of the methods of synthesis investigated in this study are: (1) a rotating parallel plate capacitor and (2) a rotating dielectric between the parallel plates of a capacitor.

Rotating Plate Method

A Fourier series in the familiar form

$$F(x) = \tfrac{1}{2} a_0 + a_1 \cos(x) + a_2 \cos(2x + a_3 \cos(3x) + \ldots \qquad (1)$$
$$+ b_1 \sin(x) + b_2 \sin(2x) + b_3 \sin(3x) + \ldots$$

may be written as

$$F(x) = \tfrac{1}{2} a_0 + A_1 \sin(x + \phi_1) + A_2) \sin(2x + \phi_2) \tfrac{1}{2} \qquad \ldots \ldots \quad (2)$$

where ϕ is a phase angle. The problem then is to obtain sine functions of integral multiple frequencies with adjustable phases and amplitudes. One method to solve this problem uses the relationship for the current to a parallel plate capacitor of variable area,

$$i = \frac{\varepsilon V}{D} \frac{dA}{dt}.$$

where ε is the permittivity, V is the applied voltage, A is the area and D is the plate separation (mks units). That is, the current is proportional to the time derivative of the plate area.

As shown in Fig. 2, a parallel plate capacitor is constructed such that the top plate is split into semi-circular halves. These halves are connected to a power source, one positive and the other negative with respect to ground. A parallel ground plate, rotated by a motor, is first under one and then the other split stationary plate. The rotating plate assumes a negative charge when it is under the positive plate and a positive charge when it is under the negative plate. It assumes some intermediate charge when it is under both halves.

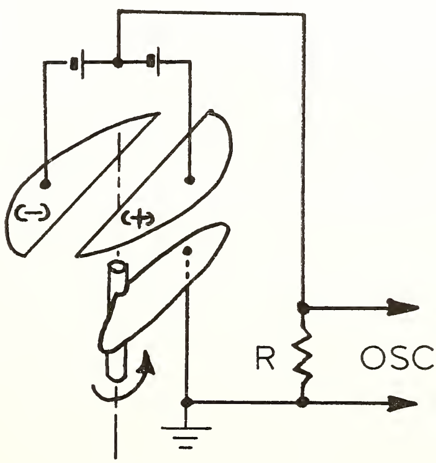

Figure 2. Split plate capacitor.

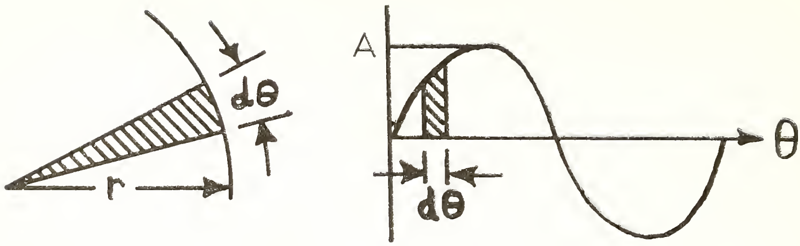

Figure 3. Equal areas.

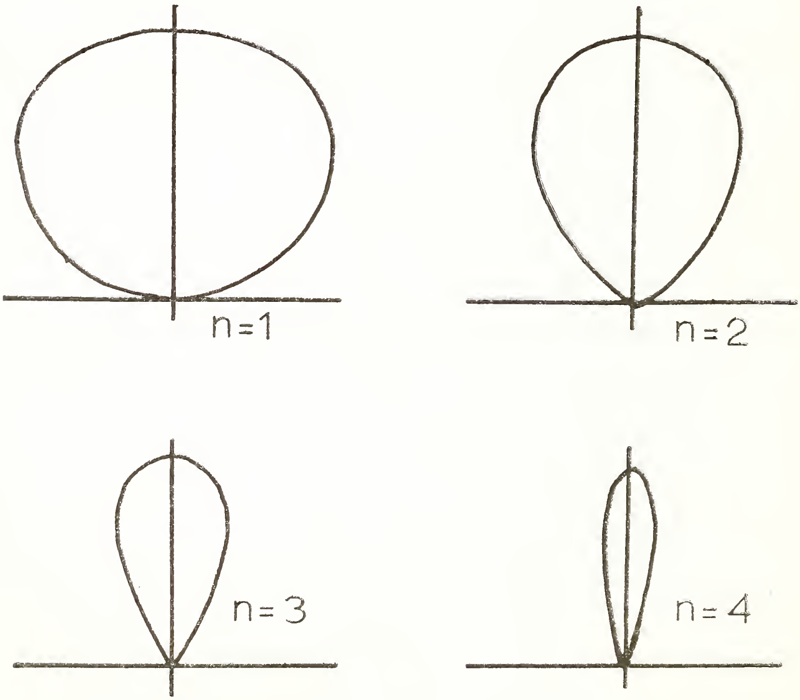

Figure 4. Base plate design.

The output signal developed across the resistor R is displayed on an oscilloscope. $\frac{dA}{dt}$ of eq (3) must be a sinusoidal function of time and so the common area between the rotating plate and one top half-plate must vary as a sine function.

The area of an infinitesimal sector in polar coordinates, as shown in Fig. 3, is proportional to $r^2\, d\theta$, and the area of an infinitesimal slice under a sine curve in cartesien coordinates is proportional to $\sin\,(\theta)\, d\theta$. Equating these two the shape of the rotating plate is described by the polar equation

$$r = \sqrt{2A \sin \theta} \tag{4}$$

The shape looks somewhat like a heart for the fundamental. See Fig. 4. For higher harmonics, the radius is proportional to the square root of the sine of $n\theta$ where n is the harmonic number and θ is the angle of rotation. It is seen in Fig. 4 that for harmonics above the fourth, the rotating plate is difficult to make.

To obtain the 2nd, 3rd, etc., harmonics, the top plate is split into 2n equal pie-shaped sectors, where n is the harmonic number. Alternate sectors are connected together; the two leads go to the plus and minus supply voltage. For a synthesizer with several harmonics, the rotating plates are fastened to a common shaft; therefore, there is a reference point for the phase angles, and the output frequencies are exact multiples of the fundamental.

To obtain a variable phase angle of the Fourier series of eq (2), the top split plates are fastened to insulators such as plexiglass, which are rotated through 360 degrees.

An experimental model employing this method was constructed for one harmonic. It gave an output very much like a sine wave. Extensive shielding was required to filter out stray capacitance and electrical noise.

Rotating Dielectric Method

Another method of generating sinusoidal functions uses a thin sheet of dielectric of the same shape as the previous method, i.e., $r \propto \sqrt{\sin (n\theta)}$. This sheet is rotated between stationary parallel plates. The dielectric increases the capacitance by a factor ϵ_r, causing the capacitance to change as a sinusoid; thus, the current through resistor R varies as a sine function.

To obtain many harmonics with adjustable phase angles, several pieces of the dielectric are connected to the same rotating shaft. The split plates may be rotated through 360° for phase control as described earlier.

Discussion

These capacitor methods were discarded in favor of a photoelectric tube method for several reasons. The precision and shielding required of the former are excessive, to name just two. The phototube method will be reported elsewhere.

Using these capacitor methods, or the better phototube method, an inexpensive, compact, and accurate synthesizer may be constructed for use in the classroom or industrial laboratory.