A HELICAL BRASS SPRING WITH CONSTANT PITCH ANGLE.

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Theoretical deductions made relating to the period of helical springs are based upon the assumption that the pitch angle (angle between the wire and a horizontal plane) is the same for all parts of the spring. If a long light spring is suspended by one end, the above condition is far from being true since the top turns will be stretched far apart due to the weight of the spring and the bottom ones will not be stretched at all. Springs can be so constructed that they will hang with their coils equally spaced. This paper presents the performance of a brass spring so corrected. The following simple method was used to correct the spring: Fasten one end of a helical spring to a rigid support so that it hangs vertically. Extend the spring using weights attached to the bottom end. Extend the spring enough to give the bottom turns a very small permanent set. The permanent set given to the coils above the bottom one will be proportional to the weight of the spring which hangs below the coils considered. If the ends are now reversed, the spring will hang with the coils equally spaced (pitch angle constant).

The theory is as follows: Let d_2 be the permanent extension of any length dx when the spring is hanging vertically just after it has received its permanent set and after the load has been removed. Assume that the permanent set is proportional to the excess stretching force P so that $P / \frac{d\rho}{dx} = K$ a constant.

Then $\frac{d_2}{dx} = \frac{P}{K} = \text{permanent set per unit length at point x.}$ Let the origin be

taken at the top, the length of the unstretched spring be L and its weight per

unit length be w. Then
$$\frac{dz}{dx} = \frac{W}{K}(1-x)$$
.

If the ends of the spring be reversed, the stretch of the spring per unit length at any point x due to its own weight will be added to the permanent stretch per unit length for the same point on the spring. It is to be shown that the sum of these two stretches will be constant for any position x.

Let the stretch per unit length due to the spring's own weight be

$$\frac{d\rho_1}{dx_1} = \frac{W}{K}(1-x_1)$$

then

$$\frac{\mathrm{d}\rho}{\mathrm{d}x} + \frac{\mathrm{d}\rho_1}{\mathrm{d}x_1} = \frac{\mathrm{W}}{\mathrm{K}} (\mathrm{l} - \mathrm{x} + \mathrm{l} - \mathrm{x}_1).$$

The position x is the same on the spring as the position when the spring is inverted.

¹Suggested by A. P. Poorman, Department of Applied Mechanics, Purdue University.

[&]quot;Proc. Ind. Acad. Sci., vol. 38, 1928 (1929)."

Therefore substitute $x_1 = 1-x$ and it will give

derelation substitute
$$x_1 = l - x$$
 and it will give $\frac{d\rho}{dx} + \frac{d\rho_1}{dx_1} = \frac{W}{K}$ a constant, which was to have been proved.

Experiments were performed to see if this were true and if such a spring were better than one not corrected. A spring made of brass, was corrected as above directed. When it was suspended from the end which was given the least permanent set, it hung with the coils all equally spaced according to theory.

Extensions, h, for an increment m of 100 grams added to the load m were taken for different loads and the periods T for each load were measured with a stop watch. Similar data were obtained with the spring inverted. The following table shows that the extensions in the two cases were approximately the same. The times of oscillation, however, were quite different as expected.

The calculated values of time from $T=2\pi\sqrt{\left(M+\frac{mo}{3}\right)\frac{h}{mg}}$ show that the spring

with coils equally spaced was much better. Errors of measurements are less than one-half of one per cent.

TABLE I. Coils Equally Spaced

	IADHE	i. Cons is	quarry opace	J.		
				M = 381.5 gms.		
Load	h/m C	scillations	Time Calc.	Time Obs.	Errer	
M.gms.	m cm/gm	No.	Sec.	Sec.	Sec.	
00		406		374.8		
100	0.13810	321	360.8	372.2	11.4	
200	0.13810	273	367.0	373.6	6.6	
300	0.13800	242	373.6	376.0	2.4	
400	0.13795	218	373.4	375.2	1.8	
500	0.13790	200	373.2	374.4	1.2	
	TABLE II	I. Coils U	nequally Space	ed		
0		406		378.6		
100	0.1378	321	360.1	374.6	14.5	
200	0.1381	273	367.0	375.2	8.2	
300	0.1380	242	373.6	377.0	3.4	
400	0.1380	218	373.4	376.0	2.6	
500	0.1380	200	373.4	375.4	2.0	

TABLE III. Showing the Difference in the Time of Oscillation

		$_{ m Time}$	$_{-}$ Time	
Load	No.	Pitch Ang.	Pitch Ang.	Difference
M (gms.)	osc.	const.	varying	of times
0	406	374.8 sec.	378.6 sec.	4.6 sec.
100	321	372.2 sec.	374.6 sec.	$2.4 \mathrm{\ sec.}$
200	273	373.6 sec.	$375.2 \mathrm{\ sec.}$	$1.6 \mathrm{\ sec.}$
300	242	376.0 sec.	377.0 sec.	$1.0 \mathrm{\ sec.}$
400	218	375.2 sec.	376.0 sec.	0.8 sec.
500	200	374.4 sec.	375.4 sec.	1.0 sec.