Some Circles Connected with the Triangle.

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In my study of the geometry of the triangle I have frequently felt the need of an available collection of the circles connected with it. So far as I know, no such collection is extant. The following list, which is by no means complete, is offered as the beginning of what it is hoped may grow into an exhaustive collection.

1. *Circumcircle.*—The circle that passes through the vertices A, B, C of the triangle. Centre at M, the point of concurrence of perpendiculars erected at the mid-points of the sides. R, the radius $= \frac{abc}{\Delta}$.

2. *Incircle.*—The circle which is tangent internally to the three sides of the triangle. Centre at I, the point of concurrence of the three internal bisectors of the angles of the triangle. r, the radius $= \frac{\Delta}{S}$.

3. *Excircles.* The three circles which are tangent externally to one side and internally to two sides of the triangle. Centres are I_1, I_2, I_3, the points of concurrence of the external bisectors of the angles with the internal bisectors of A, B, C, respectively. The radii are $r_1 = \frac{\Delta}{s - a}$, $r_2 = \frac{\Delta}{s - b}$ and $r_3 = \frac{\Delta}{s - c}$.

4. *Nine Points Circle.*—The circle which passes through the midpoints of the sides of the triangle, the feet of the perpendiculars, and the midpoints of the parts of the altitude between the orthocentre and the vertices. Centre is at F, the midpoint of IM. The radius is ½ R. It is tangent to the incircle and to each of the excircles.

5. *Brocard Circle.*—The circle whose diameter is the line joining the circumcentre M, to the symmedian point K. It passes through the two Brocard points $\Omega_1 \Omega_2$ and through the vertices of Brocard's first and second triangles. Centre at midpoint of MK.

6. *Cosine Circle.*—The circle which passes through the six points of intersection of antiparallels through K with the sides. Centre is at K (Symmedian point).

7. *Ex-Cosine Circles.*—The three circles which have K_1, K_2, K_3 (ex-symmedian points) for centres, and which pass through B, C; C, A; and A, B, respectively.

8. *The Lemoine Circle.*—The circle which passes through the six intersections of parallels through K with the sides of the triangle. The centre is at the mid-

point of MK. The centre coincides with the centre of Brocard's Circle. The radius is equal to $\frac{1}{2}\sqrt{R^2 + \rho^2}$ where ρ is the radius of the Cosine Circle. The segments cut out of the sides of the triangle by the circle are proportional to the cubes of the sides of the triangle. For this reason the circle is sometimes called the *Triplicate Ratio Circle.*

9. *Taylor's Circle.*—The circle which passes through the six projections of the vertices of the pedal triangle on the sides of the fundamental triangle.

10. *Tucker's Circles.*—The circle that passes through six points determined as follows: If on the lines KA, KB, KC, points A′, B′, C′ are taken so that $KA' : KA = KB' : KB = KC' : KC =$ a constant, then the six points above referred to are the intersections of B′C′, C′A′ and A′B′ with the sides of ABC.

The centre is at the midpoint of the line joining M and the circumcentre of A′B′C′.

The circum-, Lemoine, Cosine and Taylor Circles are particular cases of Tucker Circles.

11. *Orthocentroidal Circle.*—The circle of similitude of the circum and nine-points circle. Centre at the midpoint of HG. Radius is $\frac{1}{2}$ HG.

12. *McCay's Circles.*—The three circles which circumscribe the triangles B_2C_2G, C_2A_2G, and A_2B_2G, respectively. ($A_2B_2C_2$ is Brocard's second triangle and G is the centroid.

13. *Polar Circle.*—This is the circle with respect to which the triangle is self-conjugate. Its centre is at H. It is real when H is outside the triangle, evanescent when H is at a vertex, and imaginary when H is within the triangle.

14. ——— *Circle.*—The circle on IM as diameter. It passes through A_5, B_5, C_5, which are the midpoints of AA′, BB′, CC′, respectively. (Proceedings Indiana Academy of Sciences, 1898, page 89.)

15. *Adam's Circle.*—The circle which passes through the six points determined by the intersection with the sides of the triangle of the lines through the Gergoume Point P, parallel to I_aI_b, I_aI_c, I_cI_a, respectively. The centre is at I.

16. ——— *Circles.*—Lines through P_1, P_2, P_3 (the associated Gergoume points), parallel to the sides of $I_{1a}I_{1b}I_c$, $I_{2a}I_{2b}I_{2c}$, and $I_{3a}I_{3b}I_{3c}$, respectively, determine sets of six points on the sides which are concyclic. The centres of these three circles are at I_1, I_2, and I_3. These circles might be called the associated Adam's circles.