The Surface Tension of Liquids. By Arthur L. Foley.

Although many methods of measuring the surface tension of liquids have been proposed and used, its absolute value is not known in a single instance. Various experimenters by various methods have obtained various results; these results differing from one another in many cases by as much as fifty per cent. For instance, Quincke, for the surface tension of water at 0°C, has obtained the following results by the methods named:

1. By measuring the rise of water at a vertical wall he obtained 8.7 mgm. per mm.

2. By measuring the axis of a bubble of air in the interior of a liquid, 8.2 mgm.

3. By the rise of water in capillary tubes, 7.6 mgm.

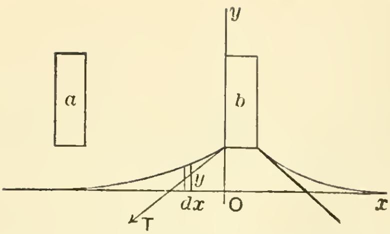
4. By measuring the size of falling drops, 6.5 mgm.

These results show an average variation of about ten per cent., and a difference between the first and last of thirty-four per cent. Many other methods have been used, but the results obtained are not more consistent than those given above. The method generally used, and that which probably gives as consistent results as any yet proposed, is the method of capillary tubes. But even if we restrict ourselves to this one method, and to the results obtained by a single experimenter, we find that they differ considerably. Let us again note the results obtained by Quincke—than whom there is no better authority upon this subject. In Wiedeman's Annalen, April, 1894, Quincke gives values ranging from 7.69 to 8.16 mgm. per mm. for different sizes of tubes made from the same specimen of Jena glass; and values from 7.8 to 8.1 for English flint glass. In the October number of the Annalen, 1894, Volkmann gives as widely different results for various specimens of glass. The age of the tube is found to influence the height to which the water rises in it. So it would seem that a better method of measuring the surface tension of liquids is greatly to be desired.

In the "Philosophical Magazine" of November, 1893, Mr. T. Proctor Hall describes some "New Methods of Measuring the Surface Tension of Liquids." Two years ago at the suggestion of Professor Michelson of the Chicago University, I undertook to repeat and to extend the investigation. In the present article, I shall confine myself to a brief statement of the results obtained by using Mr. Hall's method *c*, the maximum-weight method.[1]

[1] Philosophical Magazine, November, 1893, p. 402.

Fig 1



Let a (Fig. 1) be an end face of a rectangular parallelopiped suspended from one arm of a balance, with its lower face horizontal, and therefore parallel to the liquid surface OX. Call w' the weight of the frame (block) in this position. Lower the frame until it touches the liquid, and bring it again to the first position, as in b. The weight of the frame is now increased by the weight of the liquid raised above the level surface. As the frame is raised, the weight increases for a time then suddenly decreases, passing through a distinct maximum. Call w'' the total maximum weight. The net maximum weight is

$$w = w'' - w' = 2T\sin\alpha + \rho t y, \tag{1}$$

where $T =$ the surface tension in grams per centimeter;

$\alpha =$ the angle between the X-axis and the tangent to the liquid surface at the edge of the frame;

$t =$ the thickness of the frame;

$\rho =$ the density of the liquid;

$y =$ the height of the frame above the liquid surface;

$l =$ the length of the frame, one centimeter.

Also,

$$T\sin\alpha = \rho\int_{-\infty}^{0} y\,dx. \tag{2}$$

$$\frac{dx}{d\alpha} = \frac{T\cos\alpha}{\rho y}.$$

Placing $c^2 = \frac{T}{\rho}$, and remembering that $\frac{dy}{dx} = \tan\alpha$,

$$\frac{dy}{d\alpha} = \frac{c^2\sin\alpha}{y};$$

$$y^2 = -2c^2\cos\alpha + k.$$

When $y = 0$, $\alpha = 0$, and $k = 2c^2$.

$$\therefore y = 2c\sqrt{\frac{1-\cos\alpha}{2}} = 2c\sin\frac{\alpha}{2}.$$

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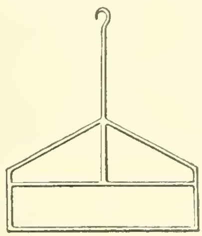
$$\cos\frac{a}{2}=\sqrt{\frac{4\,c^2-y^2}{4\,c^2}}; \tag{4}$$

$$\cos\,a=\frac{2\,c^2-y^2}{2\,c^2}. \tag{5}$$

Let us now suppose that the frame has vertical legs (as in Fig. 2) extending downward into the liquid. Let l be the length between the legs.

Fig 2



Equation (1) becomes

$$w=2T(l-t)\sin a+\rho t l y, \tag{6}$$

$$=2\,\rho c^2(l-t)\sin a+2\,lt\rho c\sin\frac{a}{2}.$$

When w is a maximum, $\frac{dw}{d\,a}=0$. Let t be very small compared with l, then

$$2\,c\cos a+t\cos\frac{a}{2}=0.$$

Eliminating a by (4) and (5), and inserting the value of c,

$$y=\sqrt{\frac{2\,T}{\rho}-\frac{t^2}{8}-t\sqrt{\frac{64}{t^2}+\frac{2\rho}{T}}}.$$

When t is small, a near approximation is

$$y=\sqrt{\frac{2\,T}{\rho}}. \tag{7}$$

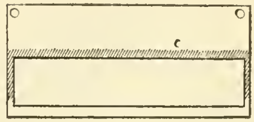
Supplying this value of y in (6), and solving for T,

$$T=\frac{w}{2\,(l-t)}+\frac{\rho l^2t^2}{4\,(l-t)^2}-\frac{lt}{4\,(l-t)^2}\sqrt{\rho^2l^2t^2+4\,w\,(l-t)\rho}. \tag{8}$$

Table II gives the value of T calculated by the above formula for mica frames varying in thickness from 0.0013 cm. to 0.02067 cm.

Mr. Hall in his investigation used glass frames (made of cylindrical glass rods) of the shape indicated in Fig. 3. He deduced for them equations correspond-

Fig 3



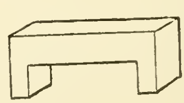
ing to (6), (7) and (8). He admits, however, that these equations are so complicated as to be almost unmanageable, and that the correction is obtained more easily by determining the constants of a frame by using frames of different length and of the same diameter, and again of the same length but of different diameters. It is very difficult indeed to make such frames, and to use them after they are made.

The chief objections to glass frames may be summed up as follows:

> The value of y, and hence the correction that must be applied to the maximum weight in order to obtain the true film weight which measures the tension, depends in a very complicated way upon the diameters of the rods of the frame.
>
> This correction forms a considerable part of the total maximum weight (see Table I.). Frames can not be made sufficiently rigid and less than 0.03 cm. in diameter. Hence the correction is at least ten per cent. of the whole.
>
> The frames are difficult to make and they require delicate handling at every stage.
>
> With cylindrical end rods the actual length of the film surface is uncertain.

It occurred to me that these troublesome corrections and inaccuracies might be partially avoided by using a different kind of frame. After experimenting with frames of various materials, among which I may mention thin sheet glass, platinum, aluminum and mica, I found that the latter offered decided advantages over glass. The general shape of the mica frame is given in Fig. 4. The frame is supported by a forked glass stem, and the method of using is exactly as with a glass frame.

Fig 4



My first frames were made by cutting the mica sheet as it lay under a steel rule upon a piece of plate glass. I afterwards had made two heavy steel plates of the exact shape of the frame desired. The inner surface of each plate was ground plane with emery dust upon plate glass. A sheet of mica was clamped between them and cut to their dimensions. The advantages of frames made in this way are:

> The steel plates are accurately ground; the frames are correspondingly regular.
>
> The mica does not split along the cut edge.
>
> The edge is of the same thickness as the plate itself; there is no bur. Very thin frames are easily made, but it is difficult to work with them when they are much less than 0.002 cm. thick.

A difficulty experienced with the mica frame, as also with those of platinum and aluminum, is that the fluid does not readily and equally wet all portions of the surface. It has a tendency to collect in drops, rendering the after-weighing uncertain. This difficulty was entirely overcome by roughing the surface (darkened in Fig. 4) of the plate near the edge by rubbing very lightly with the finest French emery paper. Both weights could then be taken again and again with a variation of only a few hundredths of a milligram.

The advantages claimed for the mica frame are as follows:

1. They are easily made, and do not require careful handling.

2. They are of even thickness, with straight edges and square corners. Hence the film length is not so uncertain as with glass frames.

3. They can be made less than one-tenth of the thickness of a glass frame, reducing the correction correspondingly. Table I gives the relative corrections for glass and mica frames, obtained by determining the maximum weight for a soap solution, and then weighing the film itself. The film weight divided by twice the length of the frame gives the surface-tension. But with many liquids it is impossible to obtain the film weight, as the film breaks immediately after it is formed. The maximum weight can be determined in almost every case, and the film weight by correction. It is evident that a slight error in the value of this correction will be lessened by reducing the total correction, as is done by using the mica frame.

TABLE I.

Kind of Frame.	l	t	w	Film Weight.	Per cent. Difference.
Glass	6.346	0.0405	0.39226	0.34100	15
Glass	7.584	0.0510	0.48283	0.40302	19
Glass	10.163	0.0620	0.65420	0.53700	21
Glass	7.475	0.0920	0.52480	0.39660	32
Mica	6.012	0.0030	0.31202	0.30697	1.6
Mica	5.301	0.0051	0.27776	0.27092	2.5
Mica	5.140	0.0079	0.27222	0.26260	3.7

A fresh solution was used in the last three measurements.

4. The correction varies directly as the thickness of the frame, Fig. 5. Observations with two frames of varying thickness are sufficient to determine the actual film weight and hence the tension.

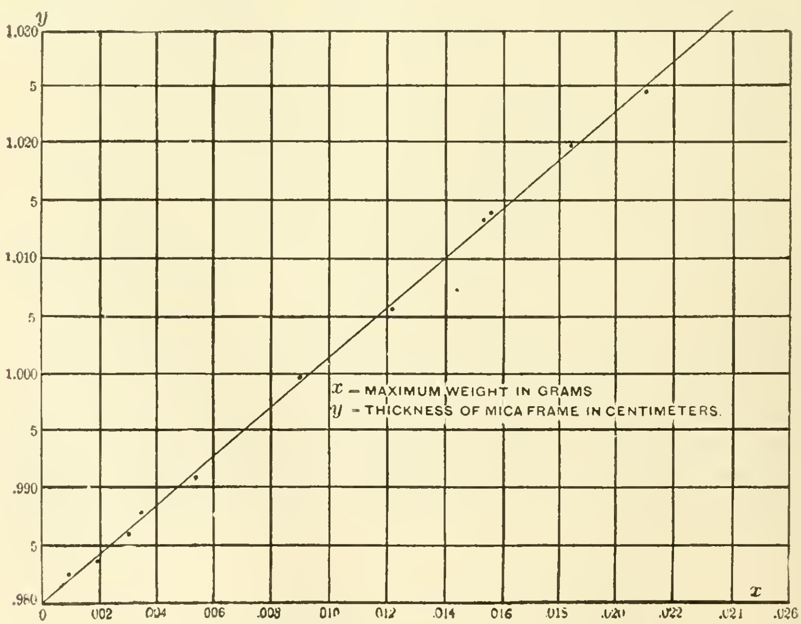


Fig 5

5. In the case of thin frames the tension can be determined at once from the maximum weight uncorrected, with results that vary less than do those obtained by the method of capillary tubes. For example, compare Table II with Table III, the latter giving selected results obtained by Quincke by the capillary tube method.[1]

TABLE II.

t	w	T by formula. $T=\frac{w}{2l}$	T by equation (8)	Temperature of Water.
0.00130 cm.	0.98260 g.	0.07396	0.07365	20°.7 C.
0.00190	0.98420	0.07408	0.07374	20°.7
0.00352	0.98791	0.07437	0.07372	20°.7
0.00516	0.99094	0.07458	0.07365	20°.7
0.00928	0.99991	0.07527	0.07352	20°.8
0.01206	1.00592	0.07572	0.07355	20°.8
0.01536	1.01358	0.07630	0.07345	20°.9
0.01828	1.01973	0.07676	0.07339	21°.0
0.02067	1.02468	0.07713	0.07332	21°.0

TABLE III.

(TEMPERATURE 18°.)

Kind of glass.	Diameter of tube.	Age of tube.	T
Common Jena glass	0.5832	0 hr.	0.07528
Common Jena glass	0.5851	24 hrs.	0.07336
English flint glass................	0.6390	2 mos.	0.07490
English flint glass................	0.5740	0 hr.	0.07411
Fusible (soft) Jena glass..........	0.6440	0 hr.	0.07258
Fusible (soft) Jena glass..........	0.9106	12 hrs.	0.07480

6. As y and t are small, a small error in the assumed value of ρ will not appreciably affect the calculated value of T, Eq. (6).

y being small, the film is much narrower than with a glass frame. Therefore there is less temperature change due to evaporation from the film surface and less absorption of gases and impurities from the air.

[1] Weidemann's Annalen, No. 5, 1894, p. 14.

7. The equations for w and y are not so complex that they can not be used. In Table II are given the values of T deduced by formula (8). It will be noted that the last frame is about sixteen times as thick as the first, yet the greatest difference in these values is but a little more than one part in two hundred. Of the results for the first four frames, the greatest difference is one part in seven thousand. The thicker frames can not be expected to give such consistent results, as the water tends to creep in between the thin layers of which the mica sheet is made up.

The Temperature Coefficient.

Previous determinations of the temperature coefficient of surface tension give results not more consistent than the values obtained for the tension itself. Brunner gives the coefficient as .14 dynes per degree, and Merian as .253 dynes. The latter result is almost double the former. Other observers give intermediate values. In view of these differences, I concluded to make a determination of the temperature coefficient by the mica frame maximum weight method. This investigation is not yet completed, so I shall not go into detail.

I am using a Troemner balance, No. 5, easily sensitive to one one-hundredth milligram. The arrangement of the balance and box or closet is very much the same as in Hall's experiment. Inside the wooden box I have a double-walled tin box, open on the side next the glass door. The space between the walls of the tin vessel (the walls being about two inches apart) may be filled with a bath to regulate the temperature of the enclosure. This temperature is obtained by reading three thermometers, placed in different positions. A rotary fan is used to equalize the temperature throughout the enclosure. It is arranged so that the water whose coefficient is to be determined is siphoned in and out of the vessel inside, without opening the door or disturbing the balance.

I have tried four methods of regulating the temperature of the enclosure. A current of air from a blower giving a very constant pressure was passed through an iron pipe heated by from one to a dozen Bunsen burners, and then through the tin box. By varying the air supply and the number of burners, a fairly constant temperature could be maintained. But I was not able to raise it above 50°. I next tried a water bath, the water being heated in a tube outside, but connected with the box—somewhat upon the principle of an incubator. I could easily maintain any desired temperature between 0° and 70°. But for higher temperatures I found that the convective circulation of the water was too slow to prevent the water in the tube from boiling. I substituted oil for water, but I was not able to extend my observations above 80°.

By far the most satisfactory method is to fill in between the walls of the tin box with mineral wool, and to use wire coils and an electric current to heat the enclosure.

In the earlier part of my work I used distilled water from the Chemical Laboratory. Subsequent tests showed that it contained considerable organic matter. I am now using water which has been distilled three times in glass; once with permanganate of potassium to remove organic matter. My observations range from 0° to 80°, and cover a period of four months.

Briefly, my conclusions are as follows:

Between 0° and 80° the temperature coefficient curve is concave toward the x axis, when we use tensions as ordinates and temperatures as abscissas. This coefficient increases with the temperature, its value being about .17 dynes.

The formula usually used to represent the tension (T) at any temperature (t°) is

$$T_{t^\circ} = T_0 - .14\ t^\circ.$$

I find that the tension can not be expressed as a linear equation, and that .14 dynes is too low for the average temperature coefficient.

Much of my work so far has been toward perfecting the method and my apparatus. I am now making some observations, using exceedingly thin mica frames, and standardized thermometers reading to one one-hundredth of a degree. For temperatures below 0° I shall use the method described by Messrs. Humphreys and Mohler in the "Physical Review," March-April, 1895. I shall endeavor to extend my observations above 100° by using the capillary tube method, the water and tubes being enclosed in an air-tight plate glass box and under whatever pressure is necessary to maintain the desired temperature without boiling the water.

Physical Laboratory, Indiana University, December, 1895.

Strains in Steam Machinery. By W. F. M. Goss.

Masses of metal when of considerable strength and weight would appear to be proof against distortion under the influence of any force which may be brought to bear upon them. We think of the *strength* of metals, but it is not often that we consider their elastic property, yet, physically speaking, nothing, probably, is more elastic than steel. A piano wire, if tightly strung, increases its length, and if loosened again it contracts. Within certain limits it behaves precisely like a spring. When force is applied it stretches, and when the force is withdrawn it