Some Skew Surfaces of the 3d and 4th Degree. C. A. Waldo.

[Abstract.]

The theory of skew ruled surfaces has been specially studied by Cremona, Cayley, Rohn and others. Rohn of Dresden has contributed several important series of models of general and fundamental character.

The object of this paper is to discuss somewhat in detail by Cartesian coördinates a family of surfaces formed by a straight line generatrix moving along two non-intersecting straight lines and a plane curve whose plane is parallel to both right lines.

Let the plane curve be $f(m, n) = Am^K + Bm^{K-1} + Cm^{K-2} + \ldots\ldots L = 0$. Let the orthogonal projections of the straight lines on the plane of this curve be axes of X and Y, and their common perpendicular the axis of Z. Let the distance from the plane of the curve to one right line directrix be pb, to the other qb, the directrix parallel to the Y axis being the more remote. In this position, by similar triangles, it is easily shown that $m : x :: pb : pb - z$, and $n : y :: qb : z - qb$. Substituting these values in $f(m, n) = 0$ we have at once a general expression for the Cartesian equation of an unlimited number of skew surfaces of this description, viz.:

$$A\left\{\frac{pbx}{pb-z}\right\}^K + B\left\{\frac{pbx}{pb-z}\right\}^K \cdot \left\{\frac{qby}{z-qb}\right\} + \ldots\ldots L = 0$$

As shown by Salmon in another way the degree of this surface is at once seen to be twice that of the directing curve or twice the product of the degrees of the directing lines of the surface.

Plane sections of this surface are in general of the 2Kth degree, but when made by the plane Z=constant, they degenerate to the Kth degree.

If the directing curve be of the 2d degree the resulting surface will be of the 4th degree unless degraded by some special position. If we take the circle as our curvilinear directrix and place it half way between the two rectilinear directrices the resulting equation will be of the form

$$\frac{b^2x^2}{(b-z)^2} + \frac{b^2y^2}{(b+z)^2} = a^2 \quad (1).$$

If the circle be replaced by the equilateral hyperbola we have

$$\frac{b^2x^2}{(b-z)^2} - \frac{b^2y^2}{(b+z)^2} = a^2 (2).$$

If the directing curve be the parabola, $x\ n^2 = 4pm$, the surface is

$$\frac{b\,y^2}{(b+z)^2} = \frac{p\,x}{b-z} \ (3),$$

a surface of the 3d degree.

In (1) if $b-a=1$, we have $x^2(1+z)^2+y^2(1-z)^2=(1+z)^2(1-z)^2$, a surface whose sections by planes perpendicular to the z axis give us between $z=0$ and $z=1$ ellipses of all possible eccentricities. A similar remark may be made of equation (2).

Among the deformations of which surface (1) is susceptible, one is worthy of special attention. If the threads representing the elements be weighted below the lower straight line directrix, and the upper directrix be then revolved until it comes into the plane of the lower directrix and the common perpendicular, the surface will gradually close up until it becomes a plane, but in every position the form of the Cartesian equation remains the same, while the axes of reference will be the equi-conjugate diameters of the ellipse cut out by the x y plane.

The Gravitational Attraction of a Homogeneous Ellipsoid of Revolution.

[Abstract.]

In this paper the following problem was discussed: Given an ellipsoid of revolution of given mass, but of variable eccentricity; find how its attraction on a particle at the end of the axis of revolution varies as the ellipsoid alters continuously from the infinitely prolate to the infinitely oblate form.

It was pointed out that this was the only case in which the expression for the attraction of a spheroid did not lead to elliptic integrals. An expression for the attraction in the above case was found by direct integration without recourse to the potential function. The integral took two forms according as the ellipsoid was prolate or oblate. The ordinary process of finding the value of the eccentricity corresponding to a maximum led to an insoluble equation. Hence the position of the maximum was approximated to by trial and interpolation. The conclusion was, that starting with the infinitely prolate form and passing through the spherical stage to the infinitely oblate form the attraction increased continuously, until that oblate stage was reached at which the axis of revolution was seventy-two hundredths of the equatorial axis, then it decreased until when the axis of revolution was fifty-one hundredths of the equatorial diameter, the attraction had fallen again to that at the spherical stage, from whence on it decreased to zero.

It was pointed out that this invalidates the common argument that the weight of a body at one of the earth's poles *must* be increased by the polar flattening.