may be given Mimulus cardinalis, Castalia Leibergii, Corydalis aurea, Polygonum Kelloggii.

Although the mountain region is very rich in Algæ Lichens Mosses and Hepatics, the conditions for work and character of our outfit made it necessary to confine our attention almost wholly to the Phanerogams and Pteridophytes, although a few lower forms were collected.

In all, ample material of about 1,000 species was brought in, which is fairly representative of the region explored.

THE APPLICATION OF MATHEMATICS IN BOTANY. By KATHERINE E. GOLDEN.

The tendency in the sciences is toward reducing results and conclusions to exactness, as far as possible, and this is as true for botany as for any of the so-called exact sciences. The tendency being toward precision, naturally the use of mathematics is becoming more general in all the sciences, in the solution of problems and the expression of results.

In physiological botany, especially, the use of mathematics is very applicable, for a great many of the principles of physiological phenomena are reducible to the principles of physics and chemistry, which are represented by mathematical formulæ, and when so represented, the conception of the phenomena is simplified, and is divested of much of the mysteriousness that attaches to it, as fundamental principles are often easier of comprehension when reduced to mathematical formulas. For instance, in studying the absorption of gases by plants, there are so many factors that enter the solution of the problem that the subject is complex to a great degree, but when it is known that the amount of gas dissolved from a mixture is proportional to the relative volume of it in the mixture multiplied by its coefficient of solubility, the quantities of gases that can be dissolved by the cell-sap are known, and a definite basis is obtained from which to start, and to take into consideration other conditions.

To show the estimate that Francis Galton* places on the laws governing the life of plants, in his work on "Natural Inheritance," in trying to arrive at some measurable characteristic by which to determine the reason for the statistical similarity shown in successive generations, he used sweet peas with which to experiment, separating them into groups ac-

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*Francis Galton. Natural Inheritance, 1889, pp. 79-82.

cording to size. The experiments were satisfactory, as they gave him the data which he sought, thus enabling him to solve the problem.

That the tendency of botanical work is in the direction of mathematical preciseness is seen in the works of Sachs, Nägeli, Wiesner and many others. Sachs* has worked out cell division in a masterly manner. By means of periclined and anticlined planes he has demonstrated the direction of the cell-divisions in a growing organ, the outline of the organ taking the form of a parabola, a hyperbola, or an ellipse. By this means he has proven that the mode of cell-division depends entirely upon the increase in volume and the configuration of the growing organ, and not upon its physiological or morphological significance. From his work he has formulated two important laws, (1) that the daughter-cells are usually equal to one another in volume, and (2) that the new cell-walls are situated at right angles to those already present.

Previous to Sachs' work it was supposed that it was possible to characterize the true morphological or phylogenetic nature of an organ by the way in which cell-division took place.

Sachs has also studied the growing apex of stems and roots so as to determine the zone of greatest growth. From the tables compiled by him there are certain facts deduced which, when the successive zones are represented by A, N, N+x, the apical zone being A, the zone of greatest growth N, and the last zone of the growing region N+x, are clearly expressed by the formula:

$$A < A+1 < A+2 \ldots < N > N+1 > N+2 \ldots > N+x.$$

The formula indicating the relation of their respective increments.

The following general expression is used by Sachs to express the relative lengths of the different tissues after isolation, where E, C, V, P, stand respectively for epidermis, cortex, vascular tissue and pith:

$$E < C < V < P > V > C > E.$$

The expression also states the relation active tension of the layers, for the greater the compression, the greater will be the length upon isolation.

Nägeli† has demonstrated the movements of bacteria in air and water. He classifies them into groups and applying the general formula for velocity $v=\sqrt{2gh}$, he has deduced the formulæ $v=\sqrt{\frac{2ghr_i}{r}}$ in which *h* is

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* J. Sachs. Arbeiten des bot. Inst. in Würzburg, 1878. On the Physiology of Plants. 1887, pp. 431-459.

† C. v. Nägeli. Untersuchungen über niedere Pilze aus dem Pflanzenphysiologischen Institut in München. 1882.

the middle vertical diameter of the body, r_i is the specific gravity of the body, and r the specific gravity of the fluid for the movement in air $v = \sqrt{2gh\frac{(r_i - r)}{r}}$ for the movement in any liquid.

Wiesner[*] has done a great deal of work in determining the application of the laws for different gases to epidermis with and without openings, at the atmospheric pressure, and pressures above and below that of the atmosphere, and with dead and living, dry and moist membranes. He has made sufficient experiments so that his conclusions, which are expressed by mathematical formulæ in many cases, are general, that is, his formula $= \frac{A}{\sqrt{d}}$ in which A represents the absorption coefficient, and d the density of the gas is general for the epidermis, free from stomata, of any plant.

An application of mathematics that one does not often see outside of the statistician's work was made by Dr. Arthur[†] in his work on pear blight. In this there was a set of determinations made as to the succulency of the fruit of the Buffum pear, so as to note the relation between the amount of moisture and the extent of the blight. After the determinations were made, calculations of the probable error in the results were also made, finding the variation in the determinations, and the extreme variation from the mean; using the figures and applying the formula, $\pm .6745\sqrt{\frac{s}{n-1}}$ in which s is the sum of the squares of the differences between each separate observation and the average of all, and n is the number of observations. This work was done to prove its correctness, as the accuracy of such work had been questioned.

The most general application of graphic mathematics is the rectilinear system of coördinates. This is so simple in the construction of diagrams and so readily understood that a great many people make use of it. Besides, one diagram will show the relation among different sets of data. Take, for example, one of Sachs' diagrams showing plant growth. The abscissæ represents increments of time, the division of the ordinates, the increments in length, the axis of abscissæ represents a certain temperature, and a certain number of the divisions of the ordinates represent a degree a temperature. Then spaces of the diagram are shaded for night. The

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[*] J. Wiesner. Versuche über den Ausgleich des Gasdruckes in den Geweben der Pflanzen, 1879. J. Weisner und H. Molisch. Untersuchungen über dis Gasbewegung in der Pflanze, 1889,

[†] J. C. Arthur. 5th Ann. Rep. N. Y. Exp. Sta., 1886, pp. 284-285.

curves of growth and temperature are represented on the same diagram, so that one can easily tell the increment of growth for a given time along with the factors of heat and light. This kind of diagram is especially valuable if the experiment be written in a language that one does not read readily, for the gist of the work can be gotten from the diagram with but little help from the text.

A great deal of mathematical work has been done in phyllotaxy. This work consisted in the first place in imagining a line proceeding from one of the older lateral members, traversing the stem to right or left, so as to include the points of insertion of all the successive lateral members in the order of their age. This line, when projected, horizontally, was called the genetic spiral, but as the line is a helix, its horizontal projection could not be a spiral.

Then in working out the law of the phyllotaxis, a series of fractions were formed, the numerator expressing the number of complete revolutions round the stem, starting from the point of insertion of a lateral organ and extending to the organ directly above it; the denominator expressing the number of joints of insertion of lateral organs passed through. It was discovered that the series of fractions expressing the most common divergences were successive convergents of the continued fraction.

$$\frac{1}{2+\frac{1}{1+\frac{1}{1+\frac{1}{1+\text{ etc.}}}}}$$

and it was supposed that a natural law had been found, but as it is necessary to construct new continued fractions for many of the divergences, this proved fallacious. But as no relation has been found to exist between the method and anything relating to plant life, the method has but little value, except from the mnemonic point of view. Work on this subject was very popular about twenty years ago, as it gave people an opportunity of proving that they knew their mathematics, it being somewhat generally supposed at the time that anyone who could do his mathematics could easily do his other work.

In the latest bulletin* issued from the Ind. Exp. Sta., the subject of which is the relation of number of eyes on the seed tuber to the product, it was found that a relation existed between the eye of the seed tuber and the number of stalks, that is, when the eyes formed an arithmetical series, the number of stalks, per unit of weight, derived from them formed an approximate hyperbolic series. To a scientific person this result means

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*J. C. Arthur. Purdue Exp. Sta. Ind., No. 42, 1892.

much, for the results are definite and given in the briefest and yet the most comprehensive manner.

When engineers publish results of experiments, they express the conditions for, and the results of, their experiments by means of mathematical formulæ as much as possible, and the tendency among botanists is to the same practice, for with the great amount of literature that is published annually, the putting the gist of the matter into the most concise and comprehensive form is becoming indispensable.

ON THE FERTILIZATION AND DEVELOPMENT OF THE EMBRYO IN SENECIO AUREUS. By D. M. MOTTIER.

DISTRIBUTION OF THE NORTH AMERICAN CACTACEÆ. By JOHN M. COULTER.

MARCHANTIA POLYMORPHA, NOT A TYPICAL OR REPRESENTATIVE LIVERWORT. By L. M. UNDERWOOD.

HOW A TENDRIL COILS. By D. T. MACDOUGAL.

FORESTRY EXHIBIT OF INDIANA AT THE COLUMBIAN EXPOSITION. By STANLEY COULTER.

NOTES ON CERTAIN PLANTS OF SOUTHWESTERN INDIANA. By JOHN S. WRIGHT.

This report is based upon about two weeks of field work done during the latter part of September, 1892, in the extreme southwestern part of the state, by D. T. MacDougal and J. S. Wright. This region is known as the "pocket" and owing to its peculiar peninsular position has an overlap of a northern and a southern flora.

Notes were made upon the distribution and condition of nearly 200 forms.